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1982 Phys. Educ. 17 209

(http://iopscience.iop.org/0031-9120/17/5/001)

Phys Educ, Vol 17, 1982 Printed in Great Britain

Measurement of refractive index using a Michelson interferometer

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The Michelson interferometer has long been a popular piece of equipment in the physics undergraduate laboratory. One possible application is the measurement of the refractive index of a thin parallel-side plate of transparent material of known thickness. (Alternatively, if the refractive index is known, the thickness of the plate may be determined.) This is accomplished by using 'white light' fringes superimposed on monochromatic fringes and is described in standard laboratory manuals (Whittle and Yarwood 1973). In this article another method for measuring the refractive index is described. Monochromatic fringes are used and plates ranging in thickness from a few micrometres to several millimetres may be measured.

Consider the plate in one arm of the Michelson interferometer. Let the ray which is normal to the mirror M make an angle of incidence φ_i with the plate (figure 1). The change in phase of the ray as it passes through the plate may be determined from considering figure 2. The phase change of the ray in going from P to Q is

$$\frac{2\pi n\,\overline{PQ}}{\lambda} = \frac{2\pi nd}{\lambda\cos\varphi_r}$$

where λ is the wavelength of the monochromatic light, n is the refractive index, d is the thickness, φ_r is the angle of refraction and $\sin\varphi_i = n\sin\varphi_r$.

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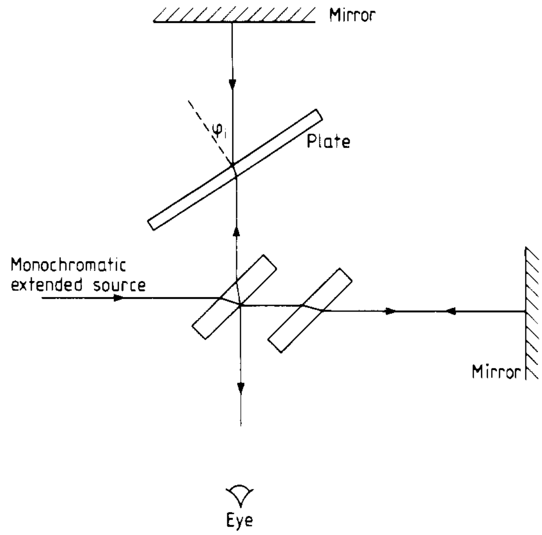


Figure 1 Transparent plate, at angle φ_i to the beam, in one arm of the Michelson interferometer

The equivalent ray in the other arm of the interferometer will pass through a corresponding air thickness $\overline{PS}$ and suffer a phase change

$$\frac{2\pi\overline{PS}}{\lambda} = \frac{2\pi d\cos(\varphi_i - \varphi_r)}{\lambda\cos\varphi_r}$$

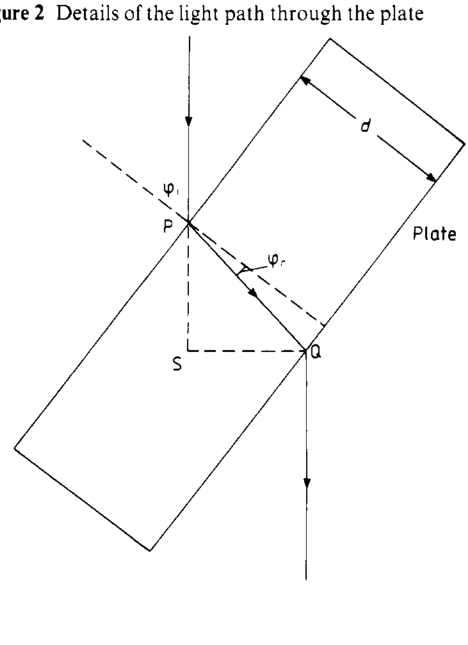
Thus the phase difference between the two rays introduced by the plate is

$$\Delta = 2\left(\frac{2\pi nd}{\lambda\cos\varphi_r} - \frac{2\pi d\cos(\varphi_i - \varphi_r)}{\lambda\cos\varphi_r}\right)$$

The factor 2 is required because the ray passes through the plate twice.

When the plate is normal to the ray ($\varphi_i = \varphi_r = 0$),

Figure 2 Details of the light path through the plate



0031-9120/82/050209+03$02.00

the phase difference between the two rays introduced by the plate is

$$\Delta_0 = 2\left(\frac{2\pi nd}{\lambda} - \frac{2\pi d}{\lambda}\right)$$

In turning the plate from the normal position through the angle φ_i, the change in the phase difference between the two rays is $(\Delta - \Delta_0)$. If this corresponds to m fringes, then $(\Delta - \Delta_0) = 2\pi m$ and

$$m = \frac{2d}{\lambda}\left[n\left(\frac{1}{\cos\varphi_r} - 1\right) + 1 - \frac{\cos(\varphi_i - \varphi_r)}{\cos\varphi_r}\right] \quad (1)$$

if n is known, d can be found from equation (1) immediately. If d is known and n unknown, it is not possible to rearrange equation (1) to give a solution for n since φ_r, which itself depends on n, is unknown. However, n can be found by using a computer and stepping n until

$$\left| m - \frac{2d}{\lambda}\left[n\left(\frac{1}{\cos\varphi_i} - 1\right) + 1 - \frac{\cos(\varphi_i - \varphi_r)}{\cos\varphi_i}\right]\right|$$

is a minimum.

An approximate value of n may be found immediately for small values of φ_i for which $\sin\varphi \approx \varphi$, $\cos\varphi \approx (1 - \varphi^2/2)$ and $\varphi_i = n\varphi_r$. Equation (1) then becomes

$$n = \frac{\varphi_i^2}{\varphi_i - m\lambda/d} \quad (2)$$

This gives a starting value for n to be used in the computer program.

Experimental procedure

The experimental arrangement is shown in figure 3. The plate was a standard glass slide of dimensions 76 mm × 25 mm × 1 mm. A holder for the plate, which allowed the plate to be rotated, was made from an inexpensive protractor. A spindle cemented to the protractor was located in a hole drilled into the base of the interferometer.

It is important that the plate be made initially normal to the beam. A check with a set-square showed that the plate was perpendicular to the base of the interferometer to within $\pm\frac{1}{2}^\circ$. Using equation (2) and the values of n, λ and d given in the next section, this gives an error in m of ± 0.04 which is negligible. To ensure that the plate was initially set at $\varphi_i = 0$, the plate was rotated in its holder 3° to 4° off normal. As the plate was moved back towards the normal position, the fringes moved one way, stopped and then moved in the reverse direction. By noting the position of the protractor for three fringes either side of the normal, the $\varphi_i = 0$ position could be found and a reference mark made on the base of the interferometer at the zero of the protractor.

With a plate thickness of 1 mm there was a shift of approximately half a fringe/degree of rotation at normal incidence rising to approximately 15 fringes/degree at $\varphi = 30°$. By attaching an arm to the protractor, some 20 cm long, it was possible to rotate the slide slowly enough to count the fringes at the higher angles of incidence.

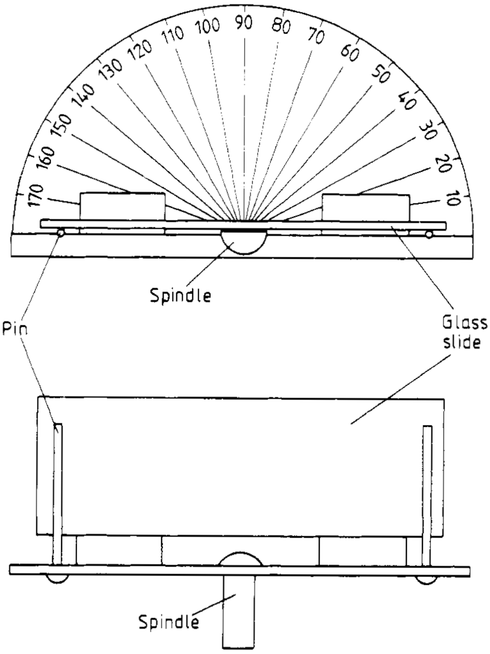


Figure 3 Holder for rotating the glass slide

The interferometer was set up for circular fringes. When the glass slide was placed in one arm at the normal position, the fringes remain circular. A theoretical analysis shows that the fringes become slightly elliptical as the slide is rotated, but with the present arrangement the effect was too small to be noticed with the unaided eye.

Results

A mercury discharge lamp with a green filter was used as the light source to give monochromatic light of wavelength 546.2 nm. The thickness of the glass slide was measured using a dial gauge and had a value of $d = 1.025 \pm 0.002$ mm. The angles φ_i from the normal through which the slide was rotated for various fringe shifts m are given in table 1. The values of the refractive index n calculated from each pair of variables using a computer program are also given. The average value of the refractive index was $n = 1.481 \pm 0.008$. This should be compared with a value of 1.482 ± 0.004 found from the 'apparent depth method' (Nelkon and Parker 1970).

Table 1 shows that the estimated precision in m and φ_i was $\pm\frac{1}{4}$ fringe and $\pm\frac{1}{4}^\circ$ respectively. The relative error in m and φ_i—and hence in n—will be large for small values of m and φ_i and a minimum value of $m = 20$ corresponding to $\varphi_i = 10.4°$ was chosen as the starting point. In calculating the average value of n given in table 1, no attempt was made to weight the individual values of n since such a detailed error analysis would be inconsistent with the unknown accuracy of the protractor.

Discussion

The values of n given in table 1 appear to show a systematic increase with increasing φ_i. Possible reasons for this include an error in setting the normal position of the plate and/or a systematic error in the angular graduations of the protractor. The systematic change in n is comparable with the random error and the precision of the experiment must be improved before it can be decided if the effect is real or not. The precision could be increased by using a properly machined holder with an angular vernier scale. Such a holder should be provided with a screw arrangement to allow the slide to be turned slowly. With such a device, readings of φ_i to within ± 5 minutes of arc could be obtained.

In deriving equation (1), the multiple reflections within the plate have not been considered. An analysis of this effect, assuming a noncomplex value of n (a perfect dielectric) shows that m is increased by $(\beta - \beta_0)/\pi$ where β is the additional phase change at angle of incidence φ_i and β_0 that at normal incidence, and

$$\tan\beta = \frac{r^2 \sin 2\delta}{1 - r^2 \cos 2\delta}$$

and

$$\tan\beta_0 = \frac{r_0{}^2 \sin 2\delta_0}{1 - r_0{}^2 \cos 2\delta_0}$$

with $\delta = 2\pi n d \cos\varphi_r/\lambda$, $\delta_0 = 2\pi n d/\lambda$, $r_0 = (n-1)/(n+1)$ and r is the reflection coefficient at the angle φ_i and is dependent on the polarisation of the light. For light polarised parallel and perpendicular to the plane of incidence,

$$r_{\parallel} = \frac{n\cos\varphi_i - \cos\varphi_r}{n\cos\varphi_i + \cos\varphi_r}$$

and

$$r_{\perp} = \frac{\cos\varphi_i - n\cos\varphi_r}{\cos\varphi_i + n\cos\varphi_r}$$

For a full discussion of the reflection of light, see Klein (1970). For the present experimental conditions the maximum value of $|(\beta - \beta_0)/\pi| < 0.04$. Since the smallest value of m measured is 20, the correction is less than 0.2%. This is negligible within the present experimental accuracy and is too small to explain the apparent systematic error.

The maximum value of φ_i may be taken as approximately 60°. After this the reflections from the plate will begin to increase significantly, the transmitted light will be decreased and the visibility of the fringes reduced. Taking $\varphi_i = 60°$ and $n = 1.5$, gives from equation (1)

$$d = 2.2\lambda m$$

The error in setting m is of the order of $\pm\frac{1}{4}$ fringe. It can thus be concluded that the resolution in the determination of the thickness d is approximately $\lambda/2$.

Table 1 *Values of fringe shift m, angle of rotation φ_i, and refractive index n*

$m \pm \frac{1}{4}$	$\varphi_i^0 \pm \frac{1}{4}°$	n	$m \pm \frac{1}{4}$	$\varphi_i^0 \pm \frac{1}{4}°$	n
20	10.4	1.474	120	24.8	1.490
40	14.7	1.470	140	26.7	1.491
60	17.9	1.473	160	28.6	1.483
80	20.6	1.473	180	30.2	1.485
100	22.9	1.477	200	31.6	1.493
		$\bar{n} \pm 3\alpha = 1.481 \pm 0.008$			

Conclusions

The experiment described gives a novel and simple method of measuring the refractive index of transparent plates using a Michelson interferometer. The experiment would be suitable in the undergraduate physics laboratory. Since it is necessary to use a computer program when determining the refractive index, students could be given the opportunity of writing their own programs.

The main teaching advantage of the present technique over the method of 'white-light' fringes is that students observe the movement of the fringes as the effective thickness of the glass in the beam increases, whereas in the 'white light' method the plate is either in or out of the beam and no fringe movement resulting from the plate is observed. The present technique is also more suitable for thick plates. Thus, in the present experiment it was sufficient to count 200 fringes for a glass plate of thickness $d = 1$ mm. The number of fringes counted in the 'white light' method as the mirror is moved would be approximately 2000 ($m = 2d(n-1)/\lambda$, see Whittle and Yarwood 1973).

As pointed out by a referee, an additional advantage of this method over the 'white light' method is that students find it difficult to find the white light fringes, especially with a plate in one arm of the interferometer. Experimentally, the present technique is much more straightforward even than the modification of the 'white light' method described by Monk (1937).

Acknowledgment

The author is grateful to Dr R S Longhurst for several helpful discussions.

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